



SECOND-YEAR OF BACHELOR OF SCIENCE MAJOR MATHEMATICS REVISED SYLLABUS ACCORDING TO CBCS NEP2020

COURSE TITLE: ALGEBRA III SEMESTER-III, W.E.F. 2024-2025

**RECOMMENDED BY THE BOARD OF STUDIES IN MATHEMATICS
AND**

APPROVED BY THE ACADEMIC COUNCIL

Devrukh Shikshan Prasarak Mandal's

Nya. Tatyasaheb Athalye Arts, Ved. S. R. Sapre Commerce, and
Vid. Dadasaheb Pitre Science College (Autonomous), Devrukh.
Tal.Sangmeshwar, Dist. Ratnagiri-415804, Maharashtra, India

Academic Council Item No: _____

Name of the Implementing Institute	:	Nya. Tatyasaheb Athalye Arts, Ved. S. R. Sapre Commerce, and Vid. Dadasaheb Pitre Science College (Autonomous), Devrukh. Tal. Sangmeshwar, Dist. Ratnagiri-415804,
Name of the Parent University	:	University of Mumbai
Name of the Programme	:	Bachelor of Science
Name of the Department	:	Mathematics
Name of the Class	:	Second Year
Semester	:	Third
No. of Credits	:	04
Title of the Course	:	Algebra -III
Course Code	:	S201MTT
Name of the Vertical in adherence to NEP 2020	:	Minor
Eligibility for Admission	:	First year Science with Maths as a minor subject in adherence to Rules and Regulations of the University of Mumbai and Government of Maharashtra
Passing Marks	:	40%
Mode of Assessment	:	Formative and Summative
Level	:	UG
Pattern of Marks Distribution for SEE and CIA	:	60:40
Status	:	NEP-CBCS
To be implemented from Academic Year	:	2024-2025
Ordinances /Regulations (if any)		

Syllabus for Second Year of Bachelor of Science in Mathematics

(With effect from the academic year 2024-2025)

SEMESTER-III

Paper No.– Mathematics Paper – I

Course Title: Algebra-III

No. of Credits - 02

Type of Vertical: Minor

COURSE CODE: S201MTT

Learning Outcomes Based on BLOOM's Taxonomy:

After completing the course, the learner will be able to...

Course Learning Outcome No.	Blooms Taxonomy	Course Learning Outcome
CLO-01	Remember	
CLO-02	Understand	Understand convergence and divergence of a sequence and series, and mean value theorems
CLO-03	Apply	Apply tests for maxima and minima to find extreme values and L'Hospital rule.
CLO-04	Analyze	Analyze convex and concave functions and graphing of a functions

Syllabus for Second Year of Bachelor of Science in Mathematics

(With effect from the academic year 2024-2025)

SEMESTER-II

Paper No.– Mathematics Paper – I

Course Title: Algebra III

No. of Credits - 02

Type of Vertical: Minor

COURSE CODE: S201MTT

Module No.	Content	No. of Lectures
Unit I Linear transformations and Matrices	<p>1. Linear transformations, representation of linear maps by matrices and effect under a change of basis, examples. Kernel and image of a linear transformation, examples.</p> <p>2. Rank-Nullity theorem and applications. Composite $S \circ T$ of linear maps $T : V \rightarrow W$ & $S : W \rightarrow U$ of f.d. real vector spaces V, W, U and matrix representation of $S \circ T$. Linear isomorphisms, inverse of a linear isomorphism. Any n-dimensional real vector space is isomorphic to \mathbb{R}^n.</p> <p>3. The following are equivalent for a linear map $T : V \rightarrow V$ of a finite dimensional real vector space :</p> <ol style="list-style-type: none"> 1. T is an isomorphism. 2. $\ker T = \{0\}$. 3. $\text{Im}(T) = V$. 	10
Unit II Matrices and Determinants	<p>1. The matrix units, row operations, elementary matrices. Elementary matrices are invertible and an invertible matrix is a product of elementary matrices. Row space and column space of a matrix, row rank and column rank of a matrix, equivalence of the row and the column rank, invariance of rank upon elementary row or column operation \mathbb{R}^n is the space of column vectors</p> $x = \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{pmatrix} \text{ where each } x_j \in \mathbb{R}, \text{ equivalence of rank of an } n \times n \text{-matrix and rank of the linear transformation } L_A : \mathbb{R}^n \rightarrow \mathbb{R}^m$ <p>$(L_A(x) = Ax \forall x \in \mathbb{R}^n)$, the dimension of solution space of the system of linear equations $Ax = 0$ equals $n - \text{rank}(A)$. The solutions of non-homogeneous systems of linear equations represented by $Ax = b$; existence of a solution when $\text{rank}(A) = \text{rank}(A, b)$; the general solutions of the system is the sum of a particular solution of the system and the solutions of the associated homogeneous system.</p> <p>2. Determinant $D(A^1, A^2)$ of order 2 and its properties:</p>	10

	<p>1. As a function of column vectors, the determinant is linear. 2. If the two columns are equal, then the determinant is equal to 0. 3. If I is the unit matrix, $I = (E^1, E^2)$, then $D(E^1, E^2) = 1$.</p> <p>Results on Determinants of order 2 :</p> <p>1. If one adds a scalar multiple of one column to the other, then the value of the determinant does not change. 2. The determinant of A is equal to the determinant of its transpose. 3. Two vectors A^1, A^2 of \mathbb{R}^2 are linearly dependent if and only if the determinant $D(A^1, A^2) = 0$. 4. Let ϕ be a function of two variables $A^1, A^2 \in \mathbb{R}^2$ such that ϕ is bilinear (i.e. ϕ is linear in each variable), $\phi(A^1, A^1) = 0$ $\forall A^1 \in \mathbb{R}^2$ and $\phi(E^1, E^2) = 1$ where $E^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $E^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are the standard unit vectors of \mathbb{R}^2, then $\phi(A^1, A^2)$ is the determinant $D(A^1, A^2)$.</p> <p>3. Determinants of order $3 \times 3, n \times n$, expansion of the determinant according to i-th row, properties of the determinant function. Results (without proof): For two $n \times n$ matrices A & B, $\text{Det}(A) = \text{Det}(A^t)$, $\text{Det}(AB) = \text{Det}(A)\text{Det}(B)$. Linear dependence and independence of vectors in \mathbb{R}^n using determinants, the existence and uniqueness of the system $Ax = b$ where A is an $n \times n$-matrix with $\det(A) \neq 0$. Cofactors and minors, adjoint $\text{adj}(A)$ of an $n \times n$-matrix A, $A \text{adj}(A) = \det(A)\text{Id}$ (without proof). A $n \times n$- real matrix is invertible if and only if $\det(A) \neq 0$ and $A^{-1} = \frac{1}{\det(A)} \text{adj}(A)$ for an invertible matrix A.</p> <p>4. Cramer's rule. Determinant as area and volume.</p>	
	Total	20

Required Previous Knowledge

Basic Mathematics Knowledge is necessary before starting to learn the course.

Access to the Course

The course is available for all the students admitted for Bachelor of Science as a Major or a minor. The students seeking admission in other disciplines may select the course as a minor considering the terms and conditions laid down by the University of Mumbai, the Government of Maharashtra, and the college, from time to time.

Forms of Assessment

The assessment of the course will be of Diagnostic, Formative and Summative type. At the beginning of the course diagnostic assessment will be carried out. The formative assessment will

be used for the Continuous Internal Evaluation whereas the summative assessment will be conducted at the end of the term. The weightage for formative and summative assessment will be 60:40. The detailed pattern is as given below.

Term End Evaluation (30 Marks)
Question Paper Pattern
Time: 1 hour

Question No.	Question Pattern	Marks
Q.1	Choose the correct alternative in each of the following.(any-10 out of 15) based on Unit I and II	20
Q.2	Long Answer Questions (A+B based on Unit I) A-Is based on theory and B is based on problems	20
Q.3	Long Answer Questions (A+B based on Unit-II) A-Is based on theory and B is based on problems	20
Total		60

The paper is evaluated out of 60 and then conversion to out of 30 is finalized.

Internal evaluation (20 Marks)

Sr. No.	Description	Marks
1	Test (30 marks converted to 10)	10
2	Active Participation in teaching learning Process	05
3	Subject related activities as assigned by the teacher	05
Total		20

Grading Scale

The grading scale used is O to F. Grade O is the highest passing grade on the grading scale, and grade F is a fail. The Board of Examinations of the college reserves the right to change the grading scale.

Reference for Unit 1 : Chapter VIII, Sections 1, 2 of **Introduction to Linear Algebra**, SERGE LANG, Springer Verlag and Chapter 4, of **Linear Algebra A Geometric Approach**, S. KUMARESAN, Prentice Hall of India Private Limited, New Delhi.

Reference for Unit 2 : Chapter VI of **Linear Algebra A geometric approach**, S. KUMARSEAN, Prentice Hall of India Private Limited, 2001 and Chapter VII **Introduction to Linear Algebra**, SERGE LANG, Springer Verlag.

Recommended Books : 1. SERGE LANG : Introduction to Linear Algebra, Springer Verlag.

2. S. KUMARESAN : Linear Algebra A Geometric approach, Prentice Hall of India Private Limited.

Additional Reference Books : References:

1. M. ARTIN : Algebra, Prentice Hall of India Private Limited.

2. K. HOFFMAN and R. KUNZE : Linear Algebra, Tata McGraw Hill, New Delhi.

3. GILBERT STRANG : Linear Algebra and its applications, International Student Edition.
4. L. SMITH : Linear Algebra, Springer Verlag.
5. A. RAMACHANDRA RAO and P. BHIMA SANKARAN : Linear Algebra, Tata McGraw Hill, New Delhi.
6. T. BANCHOFF and J. WERMER : Linear Algebra through Geometry, Springer Verlag New York, 1984.
7. SHELDON AXLER : Linear Algebra done right, Springer Verlag, New York.
8. KLAUS JANICH : Linear Algebra.
9. OTTO BRETCHER : Linear Algebra with Applications, Pearson Education.
10. GARETH WILLIAMS : Linear Algebra with Applications, Narosa Publication.