SECOND-YEAR OF BACHELOR OF SCIENCE MAJOR MATHEMATICS REVISED SYLLABUS ACCORDING TO CBCS NEP2020

COURSE TITLE: ALGEBRA-IV SEMESTER-IV, W.E.F. 2024-2025

Recommended by the Board of Studies in mathematics
And
Approved by the Academic Council

Devrukh Shikshan Prasarak Mandal's
Nya. Tatyasaheb Athalye Arts, Ved. S. R. Sapre Commerce, and
Vid. Dadasaheb Pitre Science College (Autonomous), Devrukh.
Tal.Sangmeshwar, Dist. Ratnagiri-415804, Maharashtra, India

Academic Council Item No: ______________

Name of the Implementing Institute	:	Nya. Tatyasaheb Athalye Arts, Ved. S. R. Sapre Commerce, and Vid. Dadasaheb Pitre Science College (Autonomous), Devrukh. Tal. Sangmeshwar, Dist. Ratnagiri-415804,
Name of the Parent University	:	University of Mumbai
Name of the Programme	:	Bachelor of Science
Name of the Department	:	Mathematics
Name of the Class	:	Second Year
Semester	:	Third
No. of Credits	:	04
Title of the Course	:	Algebra -IV
Course Code	:	S203MTT
Name of the Vertical in adherence to NEP 2020	:	Minor
Eligibility for Admission	:	First year Science with Maths as a minor subject in adherence to Rules and Regulations of the University of Mumbai and Government of Maharashtra
Passing Marks	:	40%
Mode of Assessment	:	Formative and Summative
Level	:	UG
Pattern of Marks Distribution for SEE and CIA	:	60:40
Status	:	NEP-CBCS
To be implemented from Academic Year	:	2024-2025
Ordinances /Regulations (if any)		

Syllabus for Second Year of Bachelor of Science in Mathematics

(With effect from the academic year 2024-2025)

SEMESTER-IV **Paper No.– Mathematics Paper – I**

Course Title: Algebra-IV **No. of Credits - 02**

Type of Vertical: Minor **COURSE CODE: S203MTT**

Learning Outcomes Based on BLOOM's Taxonomy:

After completing the course, the learner will be able to…

Course Learning Outcome No.	Blooms Taxonomy	Course Learning Outcome
CLO-01	Remember	concept and properties of Groups and Subgroups.
CLO-02	Understand	properties of Cyclic Groups and Cyclic Subgroups.
CLO-03	Apply	properties of cosets and order of a group and elements
CLO-04	Analyze	concept of homomorphism and isomorphism, automorphisms .

Syllabus for Second Year of Bachelor of Science in Mathematics

(With effect from the academic year 2024-2025)

SEMESTER-IV **Paper No.– Mathematics Paper – I**

Course Title: Algebra IV **No. of Credits - 02**

Type of Vertical: Minor **COURSE CODE: S203MTT**

Module No.	Content	No. of Lectures
Unit I **Groups and Subgroups**	(a) Definition of a group, abelian group, order of a group, finite and infinite groups. Examples of groups including (i) $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ under addition. (ii) $\mathbb{Q}^* = (= \mathbb{Q}\backslash\{0\})$, $\mathbb{R}^* = (= \mathbb{R}\backslash\{0\})$, $\mathbb{C}^* = (= \mathbb{C}\backslash\{0\})$, $\mathbb{Q}^+$(= positive rational numbers) under multiplication. (iii)$\mathbb{Z}_n$ the set of residue classes modulo n under addition. (iv) $U(n)$ the group of prime residue classes modulo n under multiplication. (v) The symmetric group S_n (vi) The group of symmetries of a plane figure. The Dihedral group D_n as the group of symmetries of a regular polygon of n sides (for $n =$ 3, 4). (vii) Klein 4-group. (viii) Matrix groups $M_{m\times n}(\mathbb{R})$ under addition of matrices, $GL_n(\mathbb{R})$ the set of invertible real matrices, under multiplication of matrices. (ix) Examples such as S', μ_n the nth roots of unity as subgroups of $\mathbb{C}^*$. **Properties such as :** 1) In a group (G,.) the following indices rules are true for all integers n, m i) $a^n a^m = a^{m+n}$ for all a in G. ii)$(a^n)^m = a^{nm}$ for all a in G. iii) $(ab)^n = a^n b^n$ for all a, b in G, whenever ab = ba. 2) In a group (G, .) the following are true i) The identity element e of G is unique. ii) The inverse of every element in G is unique. iii) $(a^{-1})^{-1} = a$ iv) $(a.b)^{-1} = b^{-1}a^{-1}$ v) If $a^2 = e$ for every a in G then (G, .) is an abelian group. vi) $(aba^{-1})^n = ab^n a^{-1}$ for every a,b in G and for every integer n. vii) If $(a.b)^2 = a^2.b^2$ for every a,b in G then (G, .) is an abelian group viii) ($\mathbb{Z}_n^*$, .) is a group if and only if n is a prime. 3) Properties of order of an element such as (n and m are integers) i) If $o(a) = n$ then $a^m = e$ if and only if $n \mid m$	10

	ii) If $o(a) = nm$ then $o(a^n) = m$ iii) If $o(a) = n$ then $o(a^n)) = \frac{n}{(n,m)}$ where (n, m) is the GCD of n and m . iv) $o(aba^{-1}) = o(b)$ and $o(ab) = o(ba)$. v) If $o(a) = n$ and $o(b) = m, ab = ba$, $(n,m) = 1$ then $o(ab) = nm$ **(b) Subgroups :** 1) Definition, necessary and sufficient condition for a non-empty set to be a Subgroup. 2) The center $Z(G)$ of a group is a subgroup. 3) Intersection of two (or a family of) subgroups is a subgroup 4) Union of two subgroups is not a subgroup in general. Union of two subgroups is a subgroup if and only if one is contained in the other. 5) If H and K are subgroups of a group G then HK is a subgroup of G if and only if $HK = KH$	
Unit II **Cyclic groups and Group homomorphism**	(a) Cyclic subgroup of a group, cyclic groups, (examples including Z, z_n *and* μ_n). (b) Properties such as (i) Every cyclic group is abelian (ii) Finite cyclic groups, infinite cyclic groups and their generators. (iii) A finite cyclic group has a unique subgroup for each divisor of the order of the group (iv) Subgroup of a cyclic group is cyclic (v) In a finite group G , $G = < a >$, if and only if $o(G) = o(a)$ (vi) If $G = < a >$ *and* $o(a) = n$ and then $G =< a^m >$ if and only if $(m, n) = 1$. (vii) If G is a cyclic group of order $p^n$and $H < G, K < G$ then prove that either $H \subseteq K$ *or* $K \subseteq H$. c) Definition of Coset and properties such as 1) If H is a subgroup of a group G and $x \epsilon G$ then prove that i) $xH = H$ if and only if $x \epsilon H$ ii) $Hx = H$ if and only if $x \epsilon H$ 2) If H is a subgroup of a group G and $x, y \epsilon G$ then prove that i) $xH = yH$ if and only if $x^{-1}y \epsilon H$ ii) $Hz = H$y if and only if $xy^{-1} \epsilon H$ 3) Lagrange's theorem and consequences such as Fermat's Little theorem, Eulers's theorem and If a group G has no nontrivial subgroups then order of G is a prime and G is Cyclic etc d) Group homomorphisms and isomorphisms, automorphisms (i) Definition.	10

	(ii) Kernel and image of a group homomorphism. (iii) Examples including inner automorphism. Properties such as 1) $f: G \to G'$ is a group homomorphism then $\ker f < G$ $f: G \to G'$ is a group homomorphism then $\ker f = \{e\}$ if and onlf if f is 1-1 $f: G \to G'$ is an isomorphism of groups then i) G is abelian if and only if G' is abelian ii) G is cyclic if and only if $G'$is cyclic.	
	Total	20

Required Previous Knowledge

Basic Mathematics Knowledge is necessary before starting to learn the course.

Access to the Course

The course is available for all the students admitted for Bachelor of Science as a Major or a minor. The students seeking admission in other disciplines may select the course as a minor considering the terms and conditions laid down by the University of Mumbai, the Government of Maharashtra, and the college, from time to time.

Forms of Assessment:-The assessment of the course will be of Diagnostic, Formative and Summative type. At the beginning of the course diagnostic assessment will be carried out. The formative assessment will be used for the Continuous Internal Evaluation whereas the summative assessment will be conducted at the end of the term. The weightage for formative and summative assessment will be 60:40. The detailed pattern is as given below.

Term End Evaluation (30 Marks)
Question Paper Pattern
Time: 1 hour

Question No.	Question Pattern	Marks
Q.1	Choose the correct alternative in each of the following.(any-10 out of 15) based on Unit I and II	20
Q.2	Long Answer Questions (A+B based on Unit I) A-is based on theory and B is based on problems	20
Q.3	Long Answer Questions (A+B based on Unit-II) A-Is based on theory and B is based on problems	20
Total		60

The paper is evaluated out of 60 and then conversion to out of 30 is finalized.

Internal evaluation (20 Marks)

Sr. No.	Description	Marks
1	Test (30 marks converted to 10)	10
2	Active Participation in teaching learning Process	05
3	Subject related activities as assigned by the teacher	05
	Total	**20**

Grading Scale

The grading scale used is O to F. Grade O is the highest passing grade on the grading scale, and grade F is a fail. The Board of Examinations of the college reserves the right to change the grading scale.

Reference for Unit 1 : Chapter 1 and 2, 1, 2 of **I.N. Herstein, Topics in Algebra,** and Chapter 2,3, of **J.B. Fraleigh, A first course in Abstract Algebra, third edition, Narosa, New Delhi**.

Reference for Unit 2 : Chapter 3 and 4,of of **I.N. Herstein, Topics in Algebra,** and Chapter 4and 5, of **J.B. Fraleigh, A first course in Abstract Algebra, third edition, Narosa, New Delhi**.

Recommended Books :

1. I.N. Herstein, Topics in Algebra, Vikas Publishing House.
2. J.B. Fraleigh, A first course in Abstract Algebra, third edition, Narosa, New Delhi.
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Additional Reference Books : References:

Additional Reference Books:

1. M. Artin: Algebra, Prentice Hall of India Private Limited.
2. K. Hoffman and R. Kunze, Linear Algebra, Tata McGraw-Hill, New Delhi.
3. G. Strang, Linear Algebra and its applications, International Student Edition.
4. L. Smith, Linear Algebra, Springer Verlag.
5. A. Ramachandra Rao and P. Bhima Sankaran, Linear Algebra, Tata McGraw-Hill, New Delhi.
6. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer Verlag New York, 1984.
7. Sheldon Axler, Linear Algebra done right, Springer Verlag, New York.
8. Klaus Janich, Linear Algebra.
9. O. Bretcher, Linear Algebra with Applications, Pearson Education.
10. G. Williams, Linear Algebra with Applications, Narosa Publication..